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Letter to the Editor

Comment on "Solutions for transversely isotopic piezoelectric infinite body, semi-finite body and bimaterial infinite body subjected to uniform ring loading and charge". *Int. J. Solids Structures* Vol. 36, No. 17, pp. 2613–2631 (1999), by Ding Haojiang, Chi Yuwei and Guo Fenglin

The recent article of Ding et al. (1999) deals with the fundamental solutions of axisymmetric problems of transversely isotropic piezoelastricity. Based on the point force solutions which were presented in their previous work (Ding and Chenbuo, 1997), the paper gives explicit expressions by superposition theorem and a series of processes of integration. The solutions for a uniform infinite body are first derived in details, then the solutions for a bimaterial infinite body are obtained by applying the same procedures. Unfortunately some mistakes exist in the Section 2. The authors have adopted some conditions to simplify the expression in the case of infinite body. However, those corresponding conditions do not hold in the case of a bimaterial infinite body. So the expressions of the final solutions presented in the Section 3 of their paper lack certain items.

My comment deals with the mistakes of the solutions of axisymmetric problems. First, the second equation of Eq. (11) of their paper should be follows instead

$$w_m = -\sum_{i=1}^{3} \frac{2\alpha_{\mathrm{im}}\bar{D}_i z_i}{l_j}\left[K(k_i) - \frac{r-r_0}{r+r_0}\Pi(d,\, k_i)\right] \tag{1}$$

Compared with the original one, Eq. (1) now contains a minus symbol. Also there is a minimal modification in the second part of the 7th equation of Eq. (12) of their paper, p_i should be replaced with o_i

$$\sigma_r = (c_{11} - c_{12)}\sum_{i=1}^{3} \frac{2\bar{D}_i}{l_i r^2}\left[\frac{p_i}{g_i}E(k_i) - (r_0^2 + z_i^2)K(k_i)\right] + \sum_{i=1}^{3} \frac{2\xi_i\bar{D}_i}{l_i}\left[\frac{o_i}{g_i}E(k_i) - K(k_i)\right] \tag{2}$$

and π should be replaced with $-\pi/2$ in Eqs. (10) and (14) of that paper,

$$\psi_i = 2D_i\left\{\frac{\pi}{2}[1 - N(r-r_0)]s_i|z| - l_i E(k_i) + \frac{r^2 - r_0^2}{l_i}K(k_i) + \frac{(r-r_0)z_i^2}{(r+r_0)l_i}\Pi(d,\, k_i)\right\} \tag{3}$$

$$\psi_0 = -2D_0\left\{\frac{\pi}{2}[1 - N(r-r_0)]s_0|z| - l_0 E(k_0) + \frac{r^2 - r_0^2}{l_0}K(k_0) + \frac{(r-r_0)z_0^2}{(r+r_0)l_0}\Pi(d,\, k_0)\right\} \tag{4}$$

Then, consider the complete expression of the integrals of Eq. (1) of Ding et al. (1999):

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$$u_r = -\sum_{i=1}^{3} \frac{2\bar{A}_i r_0 \bar{z}_{ii}}{l_{ii} r}\left[K(\bar{k}_{ii}) + \frac{r-r_0}{r+r_0}\Pi(d, \bar{k}_{ii})\right] + N(z-h)\frac{\pi r_0}{r}\left[1 + N(r-r_0)\right]\sum_{i=1}^{3}\bar{A}_i \tag{5}$$

where

$$N(r-r_0) = \begin{cases} 1, & ;\text{ when } r > r_0 \\ 0, & ;\text{ when } r = r_0 \\ -1, & ;\text{ when } r < r_0 \end{cases} \tag{6}$$

According to their previous work (Ding and Chenbuo, 1997), $\sum_1^3 A_i^P = 0$ and $\sum_1^3 A_i^Q = 0$ hold, which lead to the result $\sum_1^3 \bar{A}_i = 0$. So Eq. (5) can be simplified to the Eq. (5) of that paper. But in the case of bimaterial body, the similar results such as $\sum_1^3 \sum_1^3 \bar{A}_{ij} = 0$ can not be obtained. Unfortunately, the authors had intended to use those uncorrected conditions. The correct expression of the Eq. (20) of Ding et al. (1999) should be the follows instead:

In the region $z \geqslant 0$

$$\begin{aligned} u_r = &-\sum_{i=1}^{3} \frac{2\bar{A}_i r_0 \bar{z}_{ii}}{l_{ii} r}\left[K(\bar{k}_{ii}) + \frac{r-r_0}{r+r_0}\Pi(d, \bar{k}_{ii})\right] - \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\bar{A}_{ij} r_0 z_{ij}}{l_{ij} r}\left[K(k_{ij}) + \frac{r-r_0}{r+r_0}\Pi(d, k_{ij})\right] \\ &+ \frac{\pi r_0}{r}\left[1 + N(r-r_0)\right]\sum_{i=1}^{3}\sum_{j=1}^{3}\bar{A}_{ij} \end{aligned} \tag{7a}$$

In the region $z \leq 0$

$$u_r = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\bar{A}'_{ij} r_0 z'_{ij}}{l'_{ij} r}\left[K(k'_{ij}) + \frac{r-r_0}{r+r_0}\Pi(d, k'_{ij})\right] + \frac{\pi r_0}{r}\left[1 + N(r-r_0)\right]\sum_{i=1}^{3}\sum_{j=1}^{3}\bar{A}'_{ij} \tag{7b}$$

Eq. (7) seems that the displacement component is discontinuous at the cylindrical surface $r = r_0$. But the physical fact is that the component may be discontinuous only at the loading place ($r = r_0$, $z = h$).This means that Eq. (7) can be written in more suitable forms. By utilizing Zou et al. (1992)'s result

$$\Pi(d, k) + \Pi(k^2/d, k) - K(k) = \frac{\pi}{2}\sqrt{\frac{d}{(1+d)(k^2+d)}}\ , (0 < k < 1,\ d \neq -1) \tag{8}$$

when $z \neq 0$ and $h \neq 0$, hence $z_{ij} \neq 0$ and $z'_{ij} \neq 0$, substituting the definition of d and k(k_{ij} or k'_{ij}) into the above equation, yields

$$\left.\begin{aligned} \frac{r-r_0}{r+r_0}\Pi(d, k_{ij}) &= \frac{\pi}{2}\ \frac{l_{ij}}{z_{ij}} + \frac{r-r_0}{r+r_0}\left[K(k_{ij}) - \Pi(k_{ij}^2/d, k_{ij})\right] \\ \frac{r-r_0}{r+r_0}\Pi(d, k'_{ij}) &= -\frac{\pi}{2}\ \frac{l'_{ij}}{z'_{ij}}N(r-r_0) + \frac{r-r_0}{r+r_0}\left[K(k'_{ij}) - \Pi(k'^{\,2}_{ij}/d, k'_{ij})\right] \end{aligned}\right\},\ (r \neq r_0) \tag{9}$$

Substituting Eq. (9) into Eq. (7), yields

$$u_r = -\sum_{i=1}^{3} \frac{2\bar{A}_i r_0 \bar{z}_{ii}}{l_{ii} r}\left[K(\bar{k}_{ii}) + \frac{r-r_0}{r+r_0}\Pi(d,\,\bar{k}_{ii})\right] - \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\bar{A}_{ij} r_0 z_{ij}}{l_{ij} r}\left[\frac{2r}{r+r_0}K(k_{ij}) - \frac{r-r_0}{r+r_0}\Pi(k_{ij}^2/d,\,k_{ij})\right] + \frac{\pi r_0}{r}\sum_{i=1}^{3}\sum_{j=1}^{3}\bar{A}_{ij}\begin{pmatrix} z \geqslant 0 \\ z_{ij} \neq 0 \end{pmatrix} \tag{10a}$$

and

$$u_r = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\bar{A}'_{ij} r_0 z'_{ij}}{l'_{ij} r}\left[\frac{2r}{r+r_0}K(k'_{ij}) - \frac{r-r_0}{r+r_0}\Pi(k'^{\,2}_{ij}/d,\,k'_{ij})\right] + \frac{\pi r_0}{r}\sum_{i=1}^{3}\sum_{j=1}^{3}\bar{A}'_{ij}\begin{pmatrix} z \leqslant 0 \\ z'_{ij} \neq 0 \end{pmatrix} \tag{10b}$$

As a particular case of Eq. (7), when $z = h = 0$, we have, $u_r = 0$ when $r < r_0$ and $u_r = \frac{2\pi r_0}{r}\sum_{i=1}^{3}\sum_{j=1}^{3}\bar{A}_{ij}$ when $r > r_0$. This result coincides with the similar result in the case of transversely isotropic case obtained by Hanson and Wang (1997) who considered the problem of a ring load applied on the surface of a half-space.

In a same way, Eq. (21) of Ding et al. (1999) should be corrected as follows

In the region $z \geqslant 0$

$$w_m = -\sum_{i=1}^{3} \frac{2\alpha_{im}\bar{D}_i \bar{z}_{ii}}{\bar{l}_{ii}}\left[K(\bar{k}_{ii}) - \frac{r-r_0}{r+r_0}\Pi(d,\,\bar{k}_{ii})\right] - \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\alpha_{im}\bar{D}_{ij} z_{ij}}{l_{ij}}\left[K(k_{ij}) - \frac{r-r_0}{r+r_0}\Pi(d,\,k_{ij})\right] + \pi\left[1 - N(r-r_0)\right]\sum_{i=1}^{3}\sum_{j=1}^{3}\alpha_{im}\bar{D}_{ij} \tag{11a}$$

In the region $z \leqslant 0$

$$w_m = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\alpha'_{im}\bar{L}'_{ij} z'_{ij}}{l'_{ij}}\left[K(k'_{ij}) - \frac{r-r_0}{r+r_0}\Pi(d,\,k'_{ij})\right] + \pi\left[1 - N(r-r_0)\right]\sum_{i=1}^{3}\sum_{j=1}^{3}\alpha'_{im}\bar{L}'_{ij} \tag{11b}$$

Eq. (11) can be rewritten as follows in a more suitable form

$$w_m = -\sum_{i=1}^{3} \frac{2\alpha_{im}\bar{D}_i \bar{z}_{ii}}{l'_{ii}}\left[K(\bar{k}_{ii}) - \frac{r-r_0}{r+r_0}\Pi(d,\,\bar{k}_{ii})\right] + \pi\sum_{i=1}^{3}\sum_{j=1}^{3}\alpha_{im}\bar{D}_{ij} - \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\alpha_{im}\bar{D}_{ij} z_{ij}}{l_{ij}}\left[\frac{2r_0}{r+r_0}K(k_{ij}) + \frac{r-r_0}{r+r_0}\Pi(k_{ij}^2/d,\,k_{ij})\right]\begin{pmatrix} z \geqslant 0 \\ z_{ij} \neq 0 \end{pmatrix} \tag{12a}$$

and

$$w_m = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{2\alpha'_{im}\bar{L}_{ij} z'_{ij}}{l'_{ij}}\left[\frac{2r_0}{r+r_0}K(k'_{ij}) + \frac{r-r_0}{r+r_0}\Pi(k'^{\,2}_{ij}/d,\,k'_{ij})\right] + \pi\sum_{i=1}^{3}\sum_{j=1}^{3}\alpha'_{im}\bar{L}'_{ij}\begin{pmatrix} z \leqslant 0 \\ z'_{ij} \neq 0 \end{pmatrix} \tag{12b}$$

Other displacement components in the paper of Ding et al. (1999) are correct and need not be modified.

References

Ding, H., Chi, Y., Guo, F., 1999. Solutions for transversely isotropic piezoelectric infinite body, semi-finite body and bimaterial infinite body subjected to uniform ring loading and charge. Int. J. Solids Structures 34, 2613–2631.

Ding, H., Chen, B., 1997. On the Green's functions for two-phase transversely isotropic piezoelastric media. Int. J. Solids Structures 34, 3041–3057.

Hanson, M.T., Wang, Y., 1997. Concentrated ring loading in a full space or half-space: solutions for transversely isotropic and isotropy. Int. J. Solids Structures 34, 1379–1418.

Zou, F., Liu, Z., Zhou, H., 1992. Tables of Integrals. Astronautics Publishing House, Beijing, China (in Chinese).

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